2015-2016 1B

4. Three right triangles *ABC*, *DCE*, and *FEG* are lined up in a row and mutually connected by $\overline{AG}$ to form one big right triangle *ABG* (*Figure 4*). If $\angle BAC \cong \angle CDE \cong \angle DEC \cong \angle BCA$, AG = 12 and $m\angle BGA = 30°$, determine exactly the length *FE*.

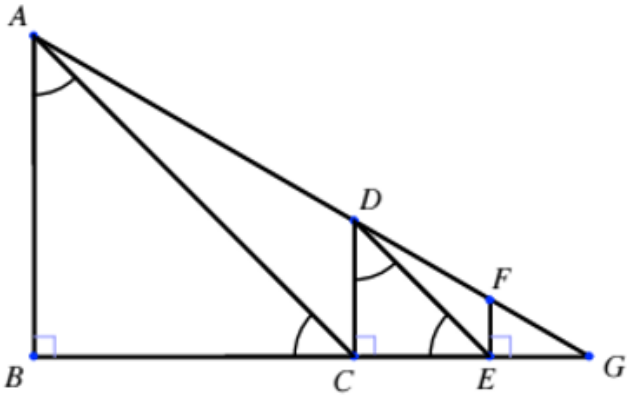


Figure 4

2014-2015 1B

In $\triangle ABC$, $m\angle A = 45°$ and $m\angle B = 30°$ as shown in *Figure 1*. If *BC* = 12, determine exactly the length *AC*.

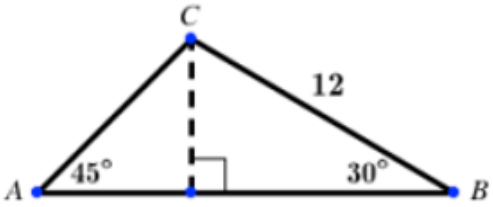


Figure 1

2014-2015 1B

Triangles *DEF* and *EGH* are both right triangles, with $m\angle D = m\angle GEH = 30°$, as shown in *Figure 3*. If *EG* = 12, determine exactly the length *DH*.

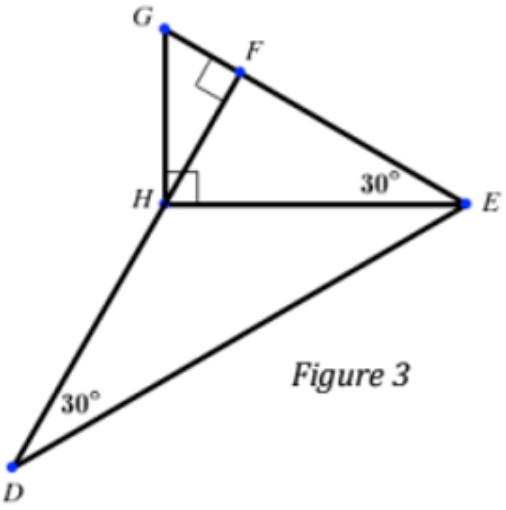
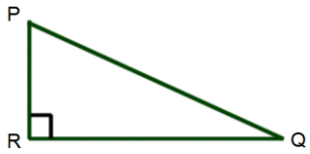


Figure 3

$\triangle PQR$ has a right angle at *R*. Points *X* and *Y* lie on $\overline{QR}$ (points *R*, *X*, *Y*, and *Q* are all distinct) such that $\angle RPX = 30°$ and $\angle XPY = \angle YPQ = 15°$. If $XY = 4$, determine exactly the value of $RX + YQ$.



SOLUTIONS

2015-2016 1B

4. Three right triangles *ABC*, *DCE*, and *FEG* are lined up in a row and mutually connected by $\overline{AG}$ to form one big right triangle *ABG* (*Figure 4*). If $\angle BAC \cong \angle CDE \cong \angle DEC \cong \angle BCA$, AG = 12 and $m\angle BGA = 30°$, determine exactly the length *FE*.

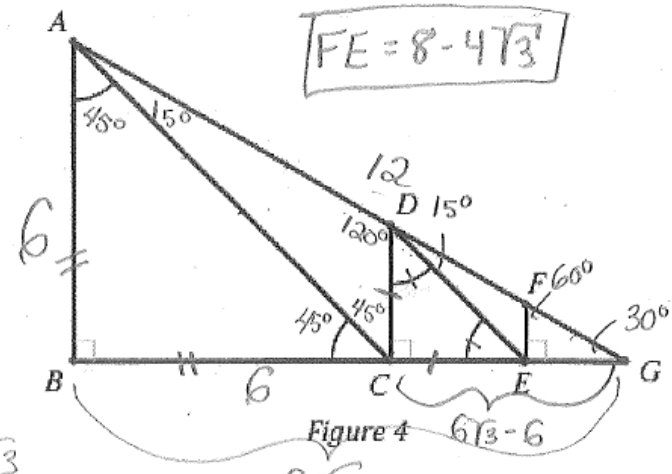


Figure 4

$FE = 8 - 4\sqrt{3}$

$CG = 6\sqrt{3} - 6$

$DC = \frac{6\sqrt{3} - 6}{\sqrt{3}}\left(\frac{\sqrt{3}}{\sqrt{3}}\right) = \frac{18 - 6\sqrt{3}}{3}$

$CE = DC = 6 - 2\sqrt{3}$

$EG = CG - CE = 6\sqrt{3} - 6 - (6 - 2\sqrt{3})$

$EG = 8\sqrt{3} - 12$

$FE = \frac{EG}{\sqrt{3}} = \frac{8\sqrt{3} - 12}{\sqrt{3}}\left(\frac{\sqrt{3}}{\sqrt{3}}\right)$

$FE = \frac{24 - 12\sqrt{3}}{3}$

2014-2015 1B

In $\triangle ABC$, $m\angle A = 45°$ and $m\angle B = 30°$ as shown in *Figure 1*. If *BC* = 12, determine exactly the length *AC*.

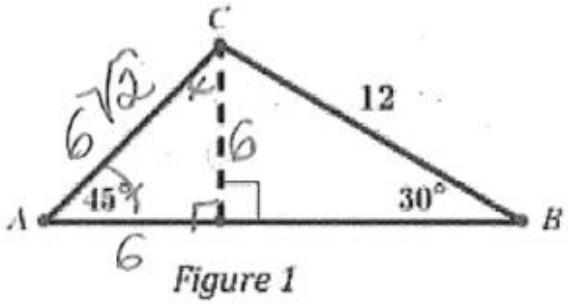


Figure 1

$6^2 + 6^2 = AC^2$

$72 = AC^2$

$AC = \sqrt{72}$

$AC = \sqrt{36 \cdot 2} = 6\sqrt{2}$

SOLUTIONS

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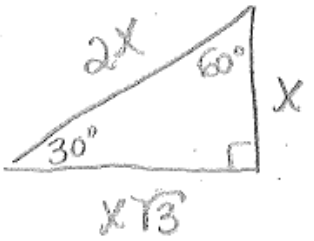
Triangles DEF and EGH are both right triangles, with $m\angle D = m\angle GEH = 30°$, as shown in *Figure 3*. If $EG = 12$, determine exactly the length DH.



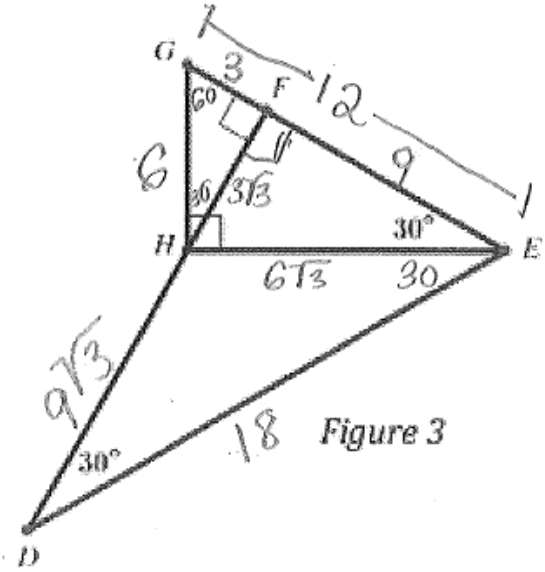


Figure 3

$\triangle PQR$ has a right angle at R. Points X and Y lie on $\overline{QR}$ (points R, X, Y, and Q are all distinct) such that $\angle RPX = 30°$ and $\angle XPY = \angle YPQ = 15°$. If $XY = 4$, determine exactly the value of $RX + YQ$.

$\boxed{2+6\sqrt{3}}$

Let $RX = a$. $\triangle PRY$ is isosceles, so $PR = a + 4$. $\triangle PRX$ is a 30-60-90 triangle, so $PR = \sqrt{3}RX \Rightarrow a + 4 = \sqrt{3}a$. Solving, $a = \dfrac{4}{\sqrt{3}-1} = 2\sqrt{3}+2$ and $PR = 2\sqrt{3}+6$. $\triangle PRQ$ is a 30-60-90 triangle with $RQ = \sqrt{3}RX$. Therefore, $RQ = \sqrt{3}(2\sqrt{3}+6) = 6+6\sqrt{3}$. So $RX + YQ = 2+6\sqrt{3}$.